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In[1]:= (* Define the deformation gradient, F, in terms of its SVD, U Σ V^T *)
U = {{u0, u3, u6}, {u1, u4, u7}, {u2, u5, u8}};
V = {{v0, v3, v6}, {v1, v4, v7}, {v2, v5, v8}};
Σ = {{σ0, 0, 0}, {0, σ1, 0}, {0, 0, σ2}};
F = U.Σ.Transpose[V];

(* Define the needed invariants (IIc is not needed) *)
Ic = σ0^2 + σ1^2 + σ2^2;
J = σ0 * σ1 * σ2;

(* Define the volume Hessian, H *)
H := {
  {0, 0, 0, 0, F[[3, 3]], -F[[2, 3]], 0, -F[[3, 2]], F[[2, 2]]},
  {0, 0, 0, -F[[3, 3]], 0, F[[1, 3]], F[[3, 2]], 0, -F[[1, 2]]},
  {0, 0, 0, F[[2, 3]], -F[[1, 3]], 0, -F[[2, 2]], F[[1, 2]], 0},
  {0, -F[[3, 3]], F[[2, 3]], 0, 0, 0, 0, F[[3, 1]], -F[[2, 1]]},
  {F[[3, 3]], 0, -F[[1, 3]], 0, 0, 0, -F[[3, 1]], 0, F[[1, 1]]},
  {-F[[2, 3]], F[[1, 3]], 0, 0, 0, 0, F[[2, 1]], -F[[1, 1]], 0},
  {0, F[[3, 2]], -F[[2, 2]], 0, -F[[3, 1]], F[[2, 1]], 0, 0, 0},
  {-F[[3, 2]], 0, F[[1, 2]], F[[3, 1]], 0, -F[[1, 1]], 0, 0, 0},
  {F[[2, 2]], -F[[1, 2]], 0, -F[[2, 1]], F[[1, 1]], 0, 0, 0, 0}
 }

(* Compute the eigenvalues of the depressed cubic *)
alpha = Ic / 3;
beta = (3 J / Ic) Sqrt[3 / Ic] ;
ε0 = 2 * Sqrt[alpha] * Cos[(1/3) * (ArcCos[beta])];
ε1 = 2 * Sqrt[alpha] * Cos[(1/3) * (ArcCos[beta] + 2 * Pi)];
ε2 = 2 * Sqrt[alpha] * Cos[(1/3) * (ArcCos[beta] + 4 * Pi)];

(* Compute the depressed cubic matrices *)
D0 = {{σ0 * σ2 + σ1 * ε0, 0, 0}, {0, σ1 * σ2 + σ0 * ε0, 0}, {0, 0, ε0^2 - σ2^2}};
D1 = {{σ0 * σ2 + σ1 * ε1, 0, 0}, {0, σ1 * σ2 + σ0 * ε1, 0}, {0, 0, ε1^2 - σ2^2}};
D2 = {{σ0 * σ2 + σ1 * ε2, 0, 0}, {0, σ1 * σ2 + σ0 * ε2, 0}, {0, 0, ε2^2 - σ2^2}};

(* Define the pseudo-cross product matrices *)
D3 := {{0, 0, 0}, {0, 0, 1}, {0, -1, 0}}
D4 := {{0, 1, 0}, {-1, 0, 0}, {0, 0, 0}}
D5 := {{0, 0, 1}, {0, 0, 0}, {-1, 0, 0}}
D6 := {{0, 0, 0}, {0, 0, 1}, {0, 1, 0}}
D7 := {{0, 1, 0}, {1, 0, 0}, {0, 0, 0}}
D8 := {{0, 0, 1}, {0, 0, 0}, {1, 0, 0}}
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(* Compute the eigenvectors of H in 3x3 matrix form *)
Q0 := U .D0.Transpose[V]
Q1 := U .D1.Transpose[V]
Q2 := U .D2.Transpose[V]
Q3 := U .D3.Transpose[V]
Q4 := U .D4.Transpose[V]
Q5 := U .D5.Transpose[V]
Q6 := U .D6.Transpose[V]
Q7 := U .D7.Transpose[V]
Q8 := U .D8.Transpose[V]

(* Define our flattening convention *)
Vec[A_] := Join[A⟦1 ;; 3, 1⟧, Join[A⟦1 ;; 3, 2⟧, A⟦1 ;; 3, 3⟧]]

(* Build flattened versions of the eigenvectors *)
q0 := Vec[Q0]
q1 := Vec[Q1]
q2 := Vec[Q2]
q3 := Vec[Q3]
q4 := Vec[Q4]
q5 := Vec[Q5]
q6 := Vec[Q6]
q7 := Vec[Q7]
q8 := Vec[Q8]

(* Define the assumptions needed by Simplify[]:
     - U and V are rotations,
     - The singular values are Reals  *)
U0 = U[[All, 1]];
U1 = U[[All, 2]];
U2 = U[[All, 3]]; UT0 = U[[1, All]];
UT1 = U[[2, All]];
UT2 = U[[3, All]];
V0 = V[[All, 1]];
V1 = V[[All, 2]];
V2 = V[[All, 3]]; VT0 = V[[1, All]];
VT1 = V[[2, All]];
VT2 = V[[3, All]];
{$Assumptions =
    U0.U1 == 0 && U0.U2 == 0 && U1.U2 == 0 && U0.U0 == 1 && U1.U1 == 1 &&
    U2.U2 == 1 && UT0.UT1 == 0 && UT0.UT2 == 0 && UT1.UT2 == 0 &&
    UT0.UT0 == 1 && UT1.UT1 == 1 && UT2.UT2 == 1 && V0.V1 == 0 && V0.V2 == 0 &&
    V1.V2 == 0 && V0.V0 == 1 && V1.V1 == 1 && V2.V2 == 1 && VT0.VT1 == 0 &&
    VT0.VT2 == 0 && VT1.VT2 == 0 && VT0.VT0 == 1 && VT1.VT1 == 1 && VT2.VT2 == 1 &&
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    Det[U] == 1 && Det[V] == 1 && σ0 ∈ Reals && σ1 ∈ Reals && σ2 ∈ Reals};

(* For each eigenvector q, we now compute Hq - λq.
   If q is indeed an eigenvector of H with eigenvalue λ,
   the resulting vector should be all zeros,
and the norm of the result should also be zero.*)
Print["If q0 is an eigenvector, this should be zero: ",
  Norm[Simplify[(H.q0) - ε0 * q0]]];
Print["If q1 is an eigenvector, this should be zero: ",
  Norm[Simplify[(H.q1) - ε1 * q1]]];
Print["If q2 is an eigenvector, this should be zero: ",
  Norm[Simplify[(H.q2) - ε2 * q2]]];
Print["If q3 is an eigenvector, this should be zero: ",
  Norm[Simplify[(H.q3) - σ0 * q3]]];
Print["If q4 is an eigenvector, this should be zero: ",
  Norm[Simplify[(H.q4) - σ2 * q4]]];
Print["If q5 is an eigenvector, this should be zero: ",
  Norm[Simplify[(H.q5) - σ1 * q5]]];
Print["If q6 is an eigenvector, this should be zero: ",
  Norm[Simplify[(H.q6) - (-σ0) * q6]]];
Print["If q7 is an eigenvector, this should be zero: ",
  Norm[Simplify[(H.q7) - (-σ2) * q7]]];
Print["If q8 is an eigenvector, this should be zero: ",
  Norm[Simplify[(H.q8) - (-σ1) * q8]]];
```

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If q0 is an eigenvector, this should be zero: 0
If q1 is an eigenvector, this should be zero: 0
If q2 is an eigenvector, this should be zero: 0
If q3 is an eigenvector, this should be zero: 0
If q4 is an eigenvector, this should be zero: 0
If q5 is an eigenvector, this should be zero: 0
If q6 is an eigenvector, this should be zero: 0
If q7 is an eigenvector, this should be zero: 0
If q8 is an eigenvector, this should be zero: 0
```